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LETTERS TO THE EDITOR

ON THE FUNDAMENTAL FREQUENCY OF A CIRCULAR PLATE SUPPORTED ON A RING

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Consider a thin, elastic plate supported by a concentric ring in the interior. The axisymmetric free vibrations of the plate have been studied by Bodine [1]. Recently, Laura *et al.* [2] produced more accurate results, again for the axisymmetric mode. However, the fundamental (lowest) frequency may not be axisymmetric. This fact was implied by a graph of Bodine [3] who plotted some higher modes. Here we shall give a discussion of this fundamental mode.

The general solution to the classical plate vibration equations in polar co-ordinates can be expressed as $w = u(r)\cos(n\theta)$, where w is the displacement, n is the number of nodal diameters, u is a linear sum of the Bessel functions $J_n(kr)$, $Y_n(kr)$, $I_n(kr)$, $K_n(kr)$ and where $k =$ (plate radius) $[(\text{density})\,(\text{frequency})^2/(\text{flexural rigidity})]^{1/4}$ is the square root of the non-dimensional frequency [4]. Let the support be at $r = b$. Let the subscript I denote the outer region $b \leqslant r \leqslant 1$ and the subscript II denote the inner region $0 \leqslant r \leqslant b$. The boundary conditions at the free edge at $r = 1$ are

$$w_{rr} + \nu\left(\frac{1}{r}\,w_r + \frac{1}{r^2}\,w_{\theta,\theta}\right) = 0, \tag{1}$$

$$\frac{\partial}{\partial r}\left(w_{rr} + \frac{1}{r}\,w_r + \frac{1}{r^2}\,w_{\theta,\theta}\right) + \frac{(1+\nu)}{r^2}\left(w_{r\theta,\theta} - \frac{w_{\theta,\theta}}{r}\right) = 0. \tag{2}$$

These conditions are equivalent to

$$u_{\mathrm{I}}''(1) + \nu[u_{\mathrm{I}}'(1) - n^2 u_{\mathrm{I}}(1)] = 0 \tag{3}$$

$$u_{\mathrm{I}}'''(1) + [n^2(\nu - 2) - 1 - \nu]\,u_{\mathrm{I}}'(1) + 3n^2 u_{\mathrm{I}}(1)] = 0. \tag{4}$$

where ν is the Poisson ratio and

$$u_{\mathrm{I}}(r) = AJ_n(kr) + BY_n(kr) + CI_n(kr) + DK_n(kr). \tag{5}$$

For the inner region we set

$$u_{\mathrm{II}}(r) = EJ_n(kr) + FI_n(kr). \tag{6}$$

The boundary conditions at the support are

$$u_{\mathrm{I}}(b) = u_{\mathrm{II}}(b) = 0, \qquad u_{\mathrm{I}}'(b) = u_{\mathrm{II}}'(b), \qquad u_{\mathrm{I}}''(b) = u_{\mathrm{II}}''(b). \tag{7–9}$$

The determinant of the coefficients of equations (3), (4), (7)–(9) is set to zero, and k is found exactly by bisection.

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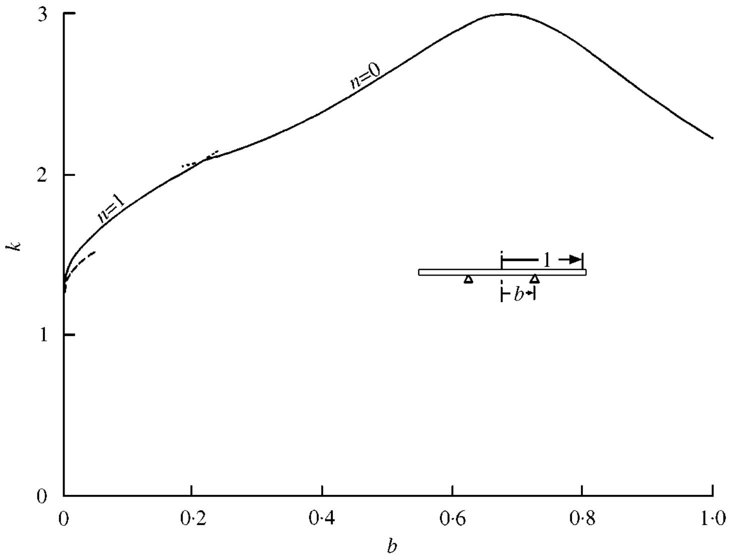

Figure 1. Fundamental frequency of a circular plate supported on a ring; – – –, from equation (10) ($\nu = 0{\cdot}3$).

TABLE 1

Fundamental frequency for $\nu = 0{\cdot}3$

b	0	0·02	0·05	0·1	0·15	0·2
k	0	1·501	1·634	1·789	1·922	2·051

The results are shown in Figure 1 for $\nu = 0{\cdot}3$. The frequency values for the axisymmetric mode ($n = 0$) agree with the exact values found in reference [2]. The maximum is $k = 3{\cdot}0005$ at $b = 0{\cdot}680$, which is also the frequency and nodal radius of the second axisymmetric mode. However, the asymmetric $n = 1$ mode gives a lower frequency when $b \leqslant 0{\cdot}211$. As $b \to 0$ the fundamental frequency drops precipitously to zero. Since the small circular support is similar to a centrally clamped condition, we can use the asymptotic form for the free-clamped plate given by Southwell [5], which in our variables is

$$k = 2\,|\ln b|^{-1/4}. \tag{10}$$

The zero-frequency at $b = 0$, of course,corresponds to the asymmetric rigid rotation about a diameter. Table 1 shows the exact fundamental frequency for small b.

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